EE 435

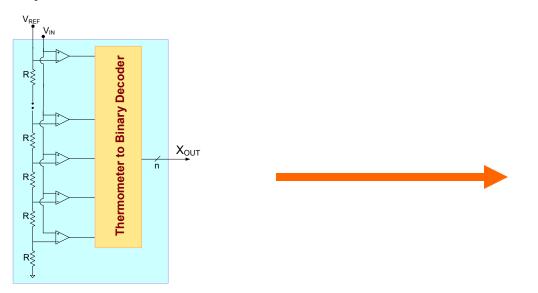
Lecture 40

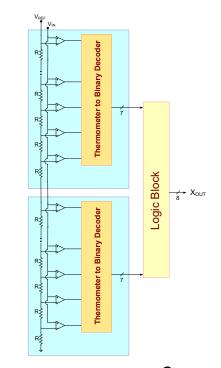
Switched-Capacitor Amplifiers and Switched-Capacitor Filters

Review from Last Lecture

How important is statistical analysis?

Example: What will be the yield if two of the 7-bit FLASH ADCs with yields of 98.8% are combined to obtain an 8-bit ADC?





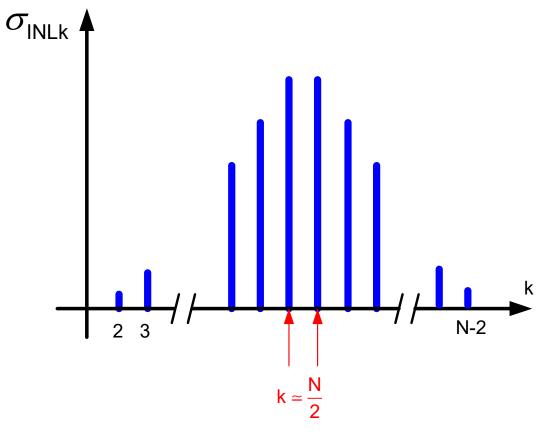
Y_{ADC}=98.8%

 $Y_{ADC} = 1.52 \bullet 10^{-6}$

- The onset of statistically-induced yield loss can be abrupt
- Intuition is not an acceptable substitute to statistical analysis
- Without statistical analysis/simulation there is a high probability that a data converter will be substantially over designed or under designed and neither is acceptable

Review from Last Lecture String DAC Statistical Performance

standard deviation of INL_k assumes a maximum variance at mid-code



It can be shown that INL_k is Gaussian and

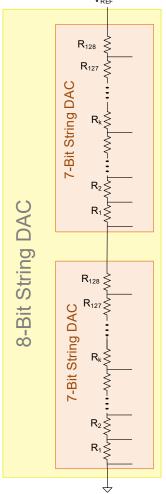
$$\sigma_{INLk\max} = \sigma_{\frac{R_R}{R_{NOM}}} \frac{\sqrt{N}}{2}$$

Review from Last Lecture

Example 3: What area is needed for obtaining a 99% yield for an 8-bit string DAC and how does that compare to the area required for a 7-bit DAC with the same yield?

For 99% yield $\sigma_z = \sigma_{\frac{R}{R_N}} \cdot \frac{\sqrt{N}}{2} = \frac{A_{\rho}}{\sqrt{A}} \cdot \frac{\sqrt{N}}{2} = 0.388$ $\frac{A_{\rho}}{\sqrt{A}} \cdot \frac{\sqrt{N}}{2} = 0.388$ $A_{\rho} = 0.1 \mu m$ N = 256 $A = 4.25 \mu m^2$

Area doubled because there are twice as many resistors and each is approximately twice as big so by adding 1-bit of resolution, the area went up by approximately a factor of 4



How important is statistical analysis?

- Statistical analysis of data converters is critical
- Some architectures are more sensitive than others to statistical variations in components
- The onset of yield loss due to statistical limitations is generally quite abrupt and can have disastrous effects if not considered as part of the design process

Recall examples where σ_{VOS} =5mV compared with σ_{VOS} =1mV

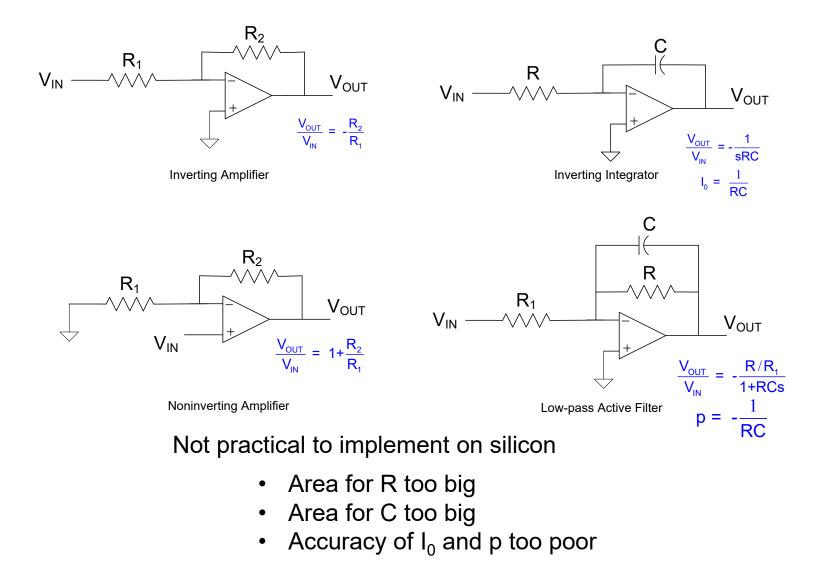
 Substantially over-designing to avoid concerns about statistical yield loss is not a practical solution since the area penalty, the speed penalty, and the power penalty are generally quite severe

For the effects of local random variations of a parameter X, generally

$$\sigma_X \propto \frac{A_0}{\sqrt{A_C}}$$

where A_C is the area of the matching critical components and A_0 is a process parameter

Some of the most basic and widely used analog circuits



But ratio accuracy can be very good (0.1% or better with good layout and appropriate area)

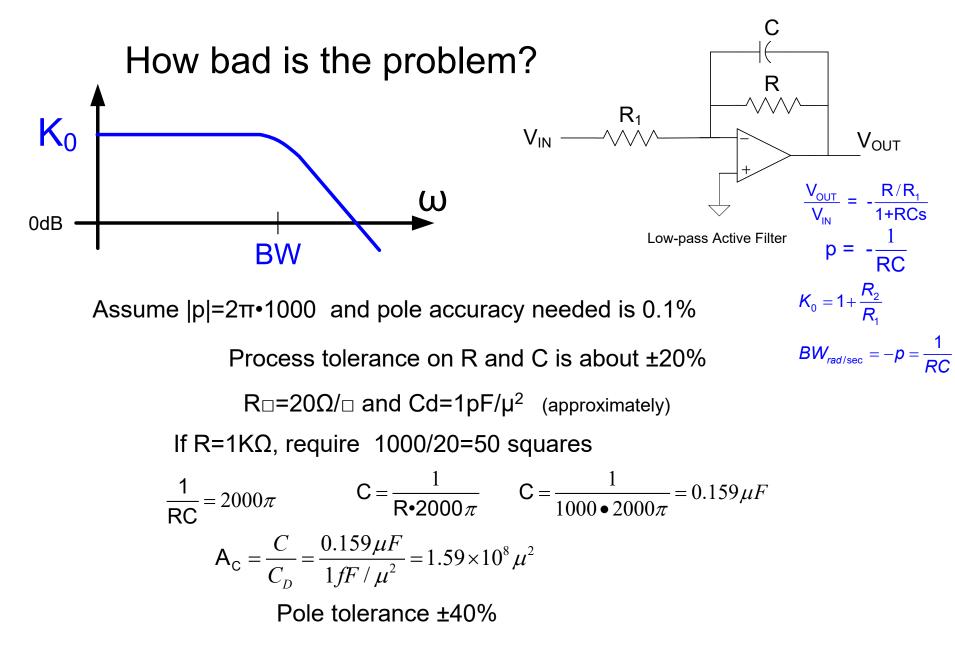
How bad is the problem?

PROCESS PARAMETERS	N+ACTV P+ACTV PO	LY PLY2 HR P	OLY2 MTL1	MTL2	UNITS
Sheet Resistance	82.7 103.2 21.	7 984 3	9.7 0.09	0.09	ohms/sq
Contact Resistance	56.2 118.4 14.	6 2	4.0	0.78	ohms
Gate Oxide Thickness	144			ang	strom
PROCESS PARAMETERS	<u>MTL3 N</u> \P	LY N_WELL	UNITS		
Sheet Resistance	0.05 8	24 815	ohms/sq		
Contact Resistance	0.78		ohms		

COMMENTS: N\POLY is N-well under polysilicon.

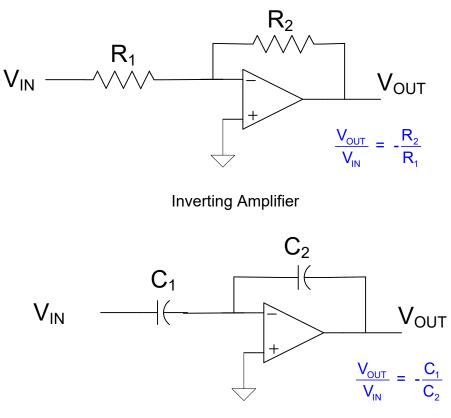
CAPACITANCE PARAMETERS	N+ACTV	P+ACTV	POLY	POLY2	M1	M2	MЗ	N_WELL	UNITS
Area (substrate)	429	721	82		32	17	10	40	aF/um^2
Area (<u>N+active</u>)			2401		36	16	12		aF/um^2
Area (<u>P+active</u>)			2308						aF/um^2
Area (poly)				864	61	17	9		aF/um^2
Area (poly2)					53				aF/um^2
Area (metall)						34	13		aF/um^2
Area (metal2)							32		aF/um^2
Fringe (substrate)	311	256			74	58	39		aF/um
Fringe (poly)					53	40	28		aF/um
Fringe (metall)						55	32		aF/um
Fringe (metal2)							48		aF/um
Overlap (<u>N+active</u>)			206						aF/um
Overlap (<u>P+active</u>)			278						aF/um

 $R_{\Box}=21.7\Omega/_{\Box}$ and $Cd=0.864 fF/\mu^2$



Both are orders of magnitude unacceptable !

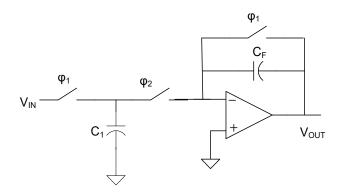
An amplifier alternative ?

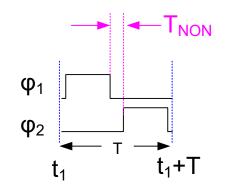


Inverting Amplifier

- Capacitor version is area effective and can have very good accuracy
- The node between C₁ and C₂ is a floating node if the Op Amp has a MOS differential pair at the input
- But if we get any charge on the intermediate node there is no way to get it off

An amplifier alternative ?:





 Φ_1 and Φ_2 are nonoverlapping clocks

During Φ_1

 C_1 is charged to V_{IN} and stores charge $Q_1 = C_1 V_{IN}$

 C_F is discharged and $V_{OUT}=0$

During Φ_2

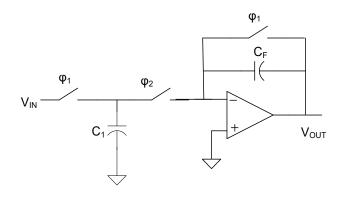
 $\rm C_1$ is discharged but charge is transferred to $\rm C_F$

$$Q_2 = -Q_1$$
 and $V_{OUT} = Q_2/C_F$

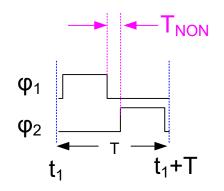
Substituting for Q₁ we obtain $V_{OUT} = -\frac{C_1}{C_r}V_{IN}$

Serves as a voltage amplifier with output valid during φ_2

An amplifier alternative !



 $V_{OUT} = -\frac{C_1}{C_2}V_{IN}$

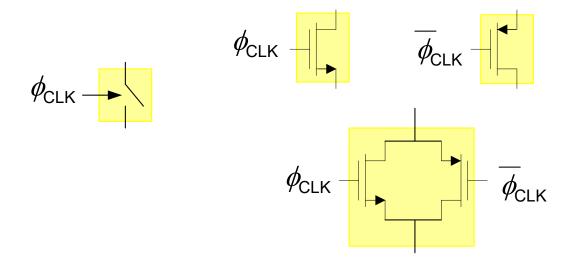


 Φ_1 and Φ_2 are nonoverlapping clocks

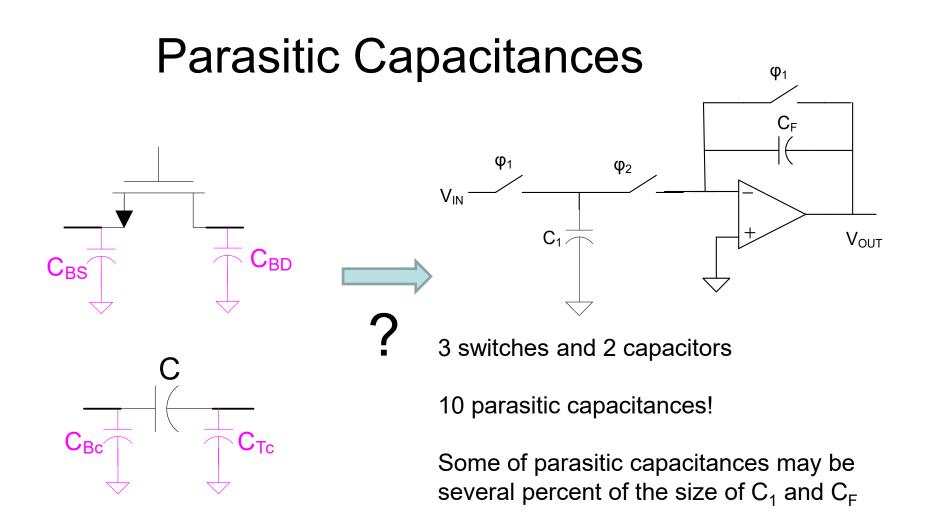
- Many applications only need amplifier output at discrete points in time
- Accuracy can be very good
- Area can be very small

But, what about the switches?

Switches for SC Circuits



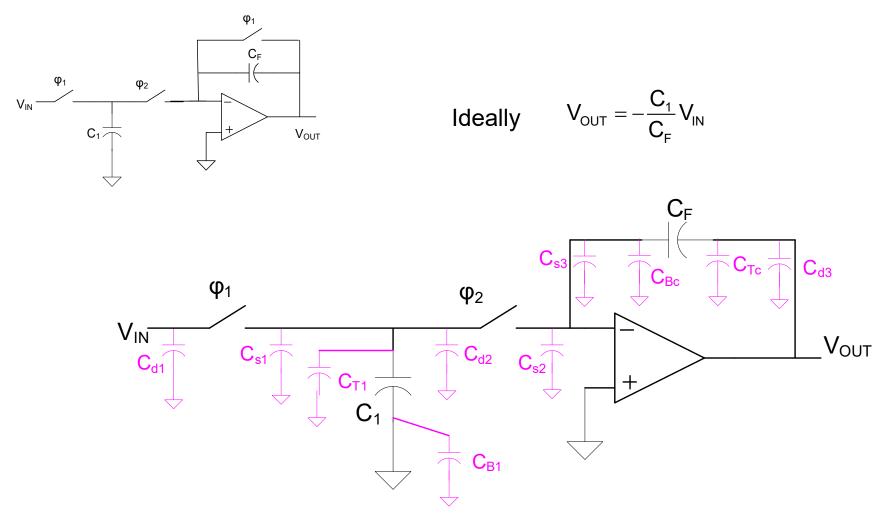
- Often a single MOS transistor is adequate (either n-ch or p-ch)
- Sometimes need transmission-gate switch (parallel n-ch and p-ch)
- Switches work very well and can be very small but must manage their R_{ON}



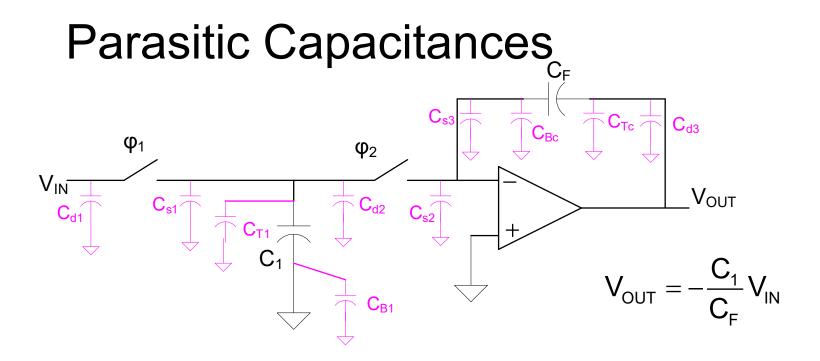
And parasitic capacitors do not match very well

And parasitic capacitances may be highly nonlinear (junction capacitors)

Parasitic Capacitances



10 parasitic capacitances!



 C_1/C_F can be very precisely controlled with appropriate layout and area allocation

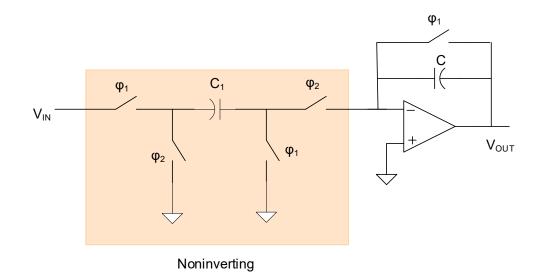
If op amp is ideal, C_{d1} , C_{B1} , C_{s2} , C_{s3} , C_{Bc} , C_{Tc} and C_{d3} do not affect charge transfer !

But C_{s1}, C_{T1}, C_{d2} are all in parallel with C_1 and all transfer charge

$$V_{OUT} = -\frac{C_1 + (C_{s1} + C_{T1} + C_{d2})}{C_r} V_{IN}$$

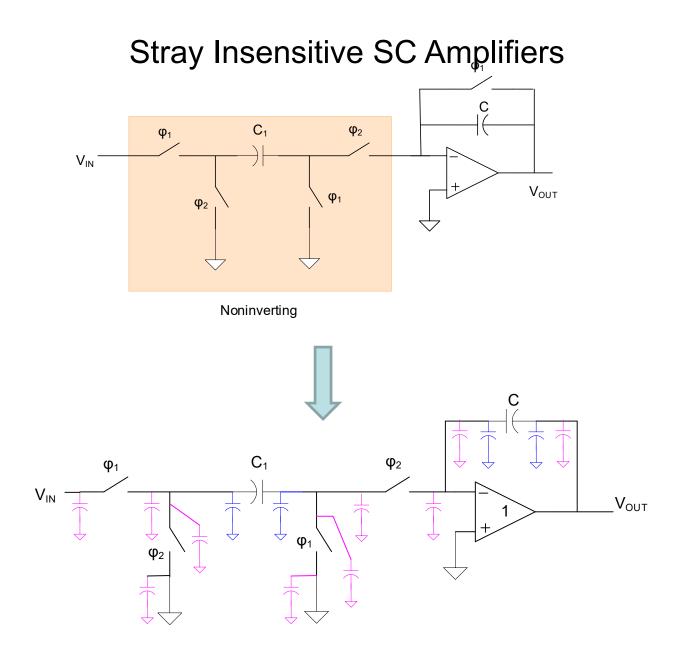
Parasitic capacitances not accurately controlled and dramatically degrade matching!

Stray Insensitive SC Amplifiers



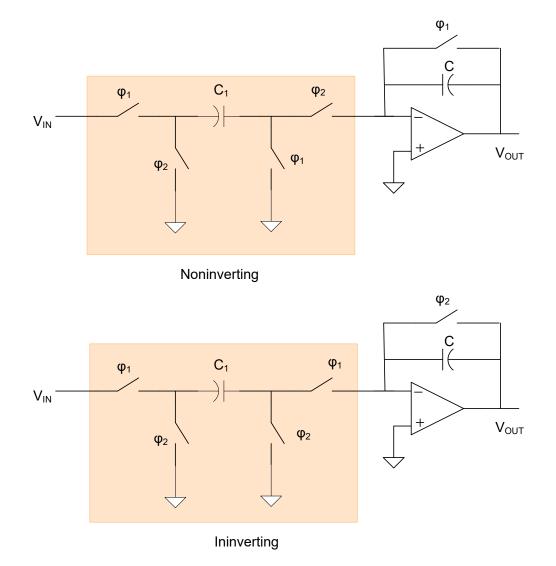
Another SC Amplifier with even more switches !

Increased to 14 diffusion parasitic capacitances



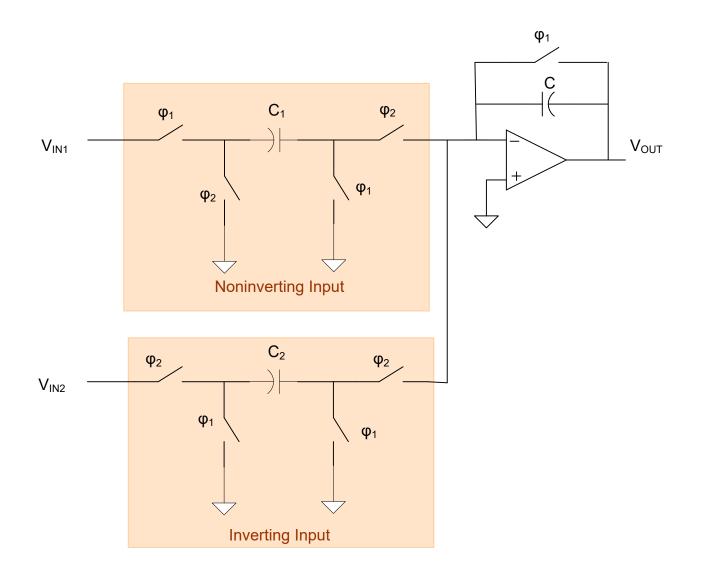
Can show that all 14 diffusion parasitic capacitances do not affect gain !!

Stray Insensitive SC Amplifiers

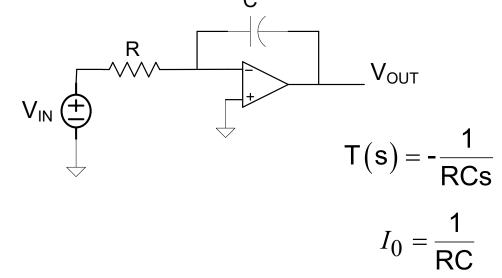


Can show that all diffusion parasitic capacitances do not affect gain

Summing amplifier inputs either inverting or noninverting can be easily obtained



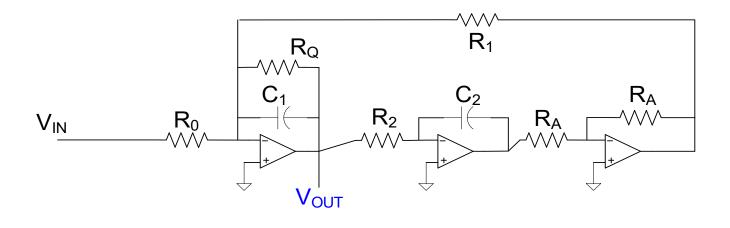
Consider the $\mathop{\text{Basic}}_{\rm c}$ Integrator



Key performance of integrator (and integrator-based filters) is determined by the integrator time constant ${\sf I}_0$

Precision of time constants of a filter invariably determined by precision of I_0

Integrator-Based Filters:

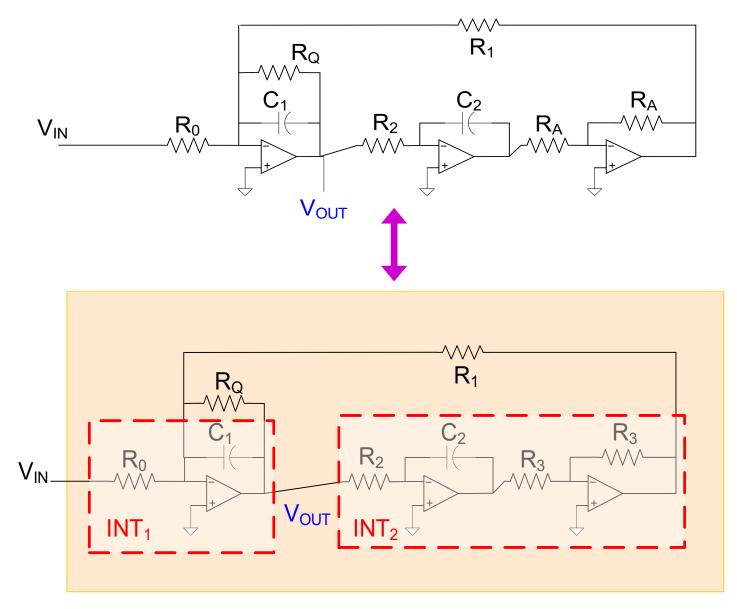


$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_0 C_1} \frac{s}{s^2 + s \left(\frac{1}{R_0 C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

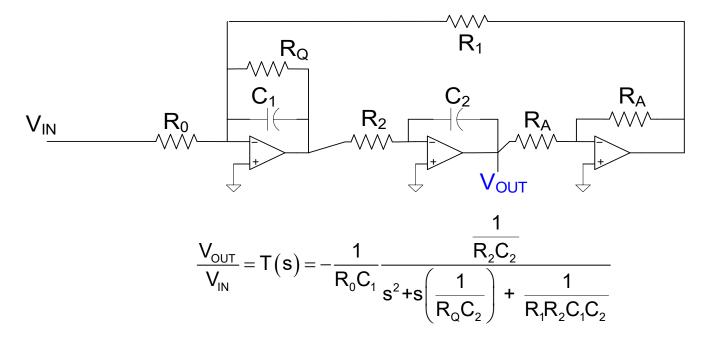
Second-order Bandpass Filter

Denote as a two-integrator-loop structure

Integrator-Based Filters:



Integrator-Based Filters:

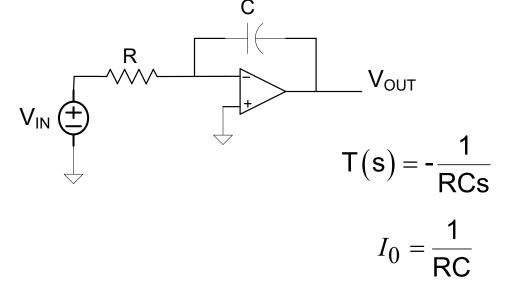


Second-order Lowpass Filter

Denote as a two-integrator-loop structure

- Any filter transfer function can be implemented with integrators and summers
- Some of the best known filter structures are based upon integrators and summers
- Accuracy of RC products is critical in the design of good filters

Consider the Basic Integrator

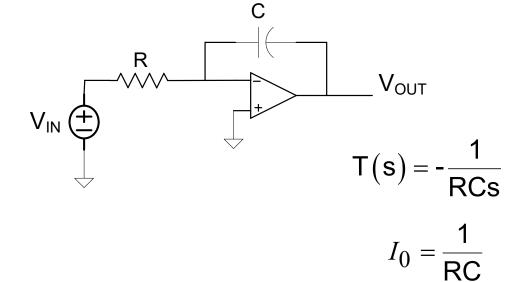


Accurate control of I_0 is required to build good filters !

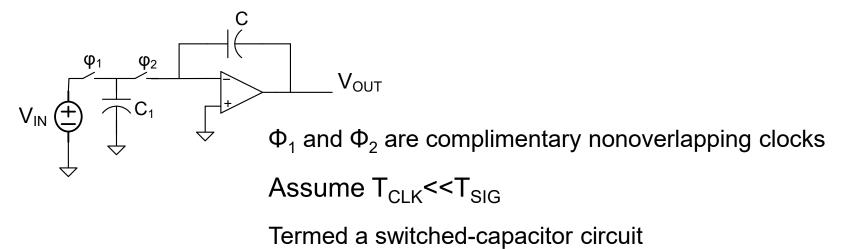
- 1. Accuracy of R and C difficult to accurately control particularly in integrated applications (often 2 or 3 orders of magnitude to variable)
- 2. Size of R and C unacceptably large if I₀ is in audio frequency range (2 or 3 orders of magnitude too large)
- 3. Amplifier GB limits performance

Appears to be Incredible Challenge to Building Filters on Silicon!

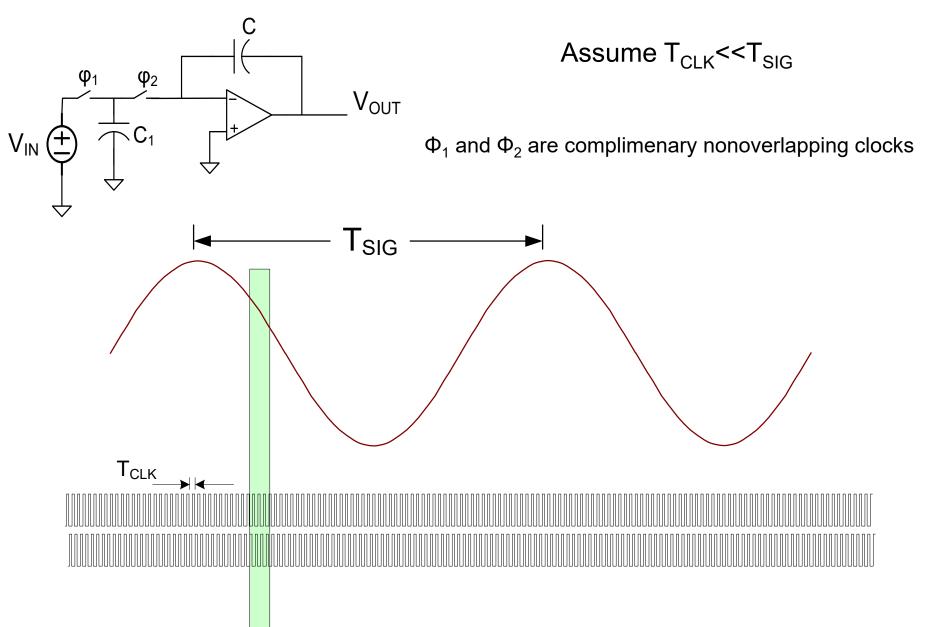
Integrator Design Issues



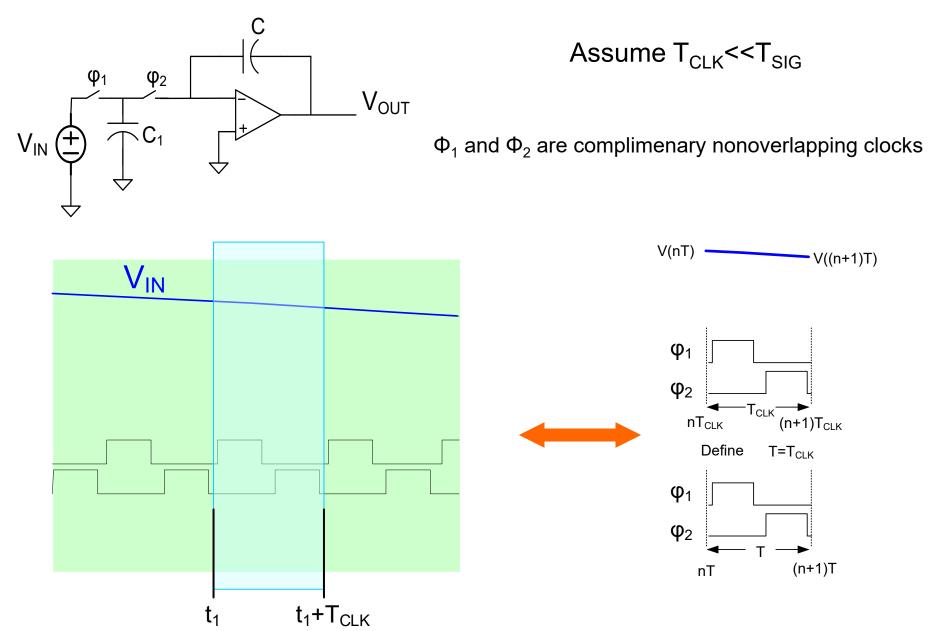
Consider:



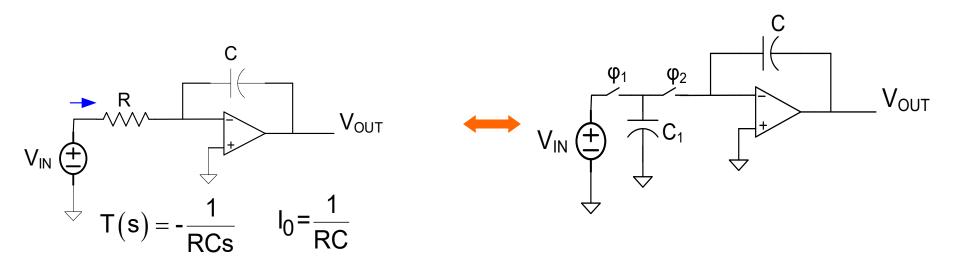
Consider the Switched-Capacitor Circuit



Consider the Switched-Capacitor Circuit

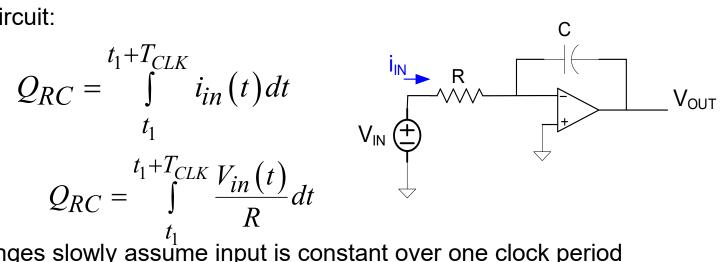


Compare the performance of the following two circuits



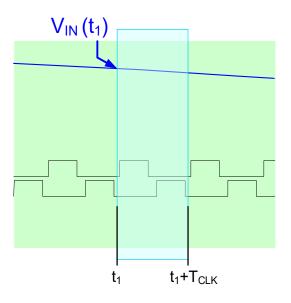
Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at arbitrary time t_1

For the RC circuit:



Since V_{in} changes slowly assume input is constant over one clock period

$$Q_{RC} \simeq \int_{t_1}^{t_1 + T_{CLK}} \frac{V_{in}(t_1)}{R} dt$$
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right]_{t_1}^{t_1 + T_{CLK}} 1 dt$$
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right]_{t_1}^{t_1 + T_{CLK}} 1 dt$$



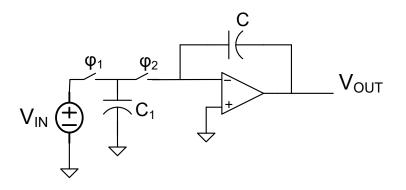
Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

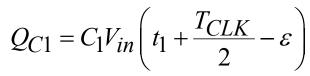
For the RC circuit:

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

Observe that a resistor "transfers" charge proportional to $V_{\mbox{\scriptsize in}}$ in a short interval of $T_{\mbox{\scriptsize CLK}}$

For the SC circuit





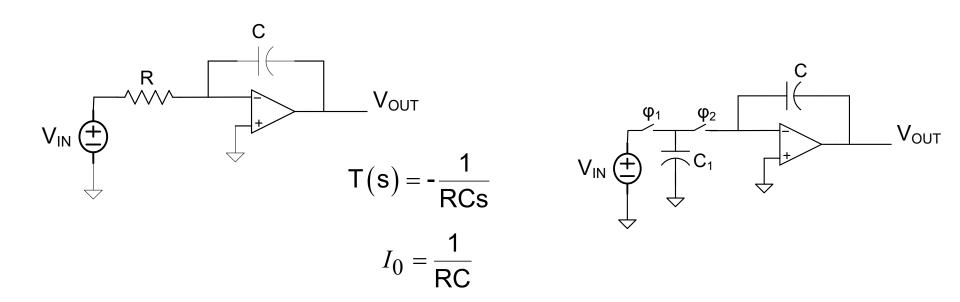
Since V_{in}(t) is slowly varying

 $Q_{C1}\simeq C_{1}V_{in}\left(t_{1}\right)$

But this is the charge that will be transferred to C during phase Φ_2

 $Q_{SC} \simeq C_1 V_{in}(t_1)$

Observe that the SC circuit also transfers charge proportional to V_{in} in short intervals of length T_{CLK}



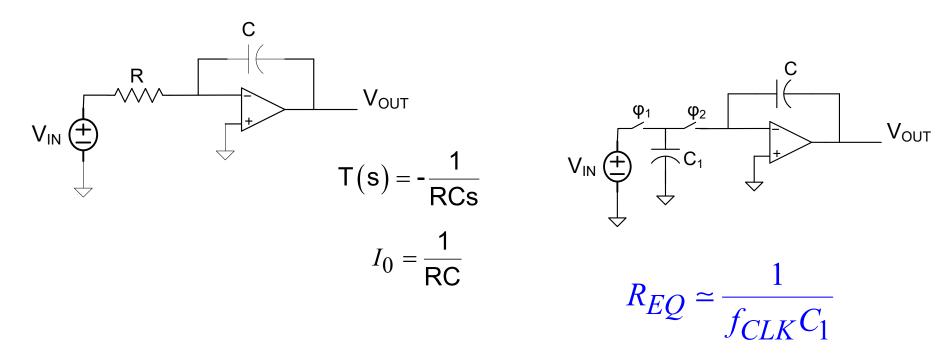
Comparing the two circuits

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

$$Q_{SC} \simeq C_1 V_{in}(t_1)$$

Equating charges since both proportional to $V_{in}(t_1)$

$$C_{1} \simeq \left[\frac{1}{R}\right] T_{CLK}$$
$$R_{EQ} \simeq \frac{1}{f_{CLK}C_{1}}$$



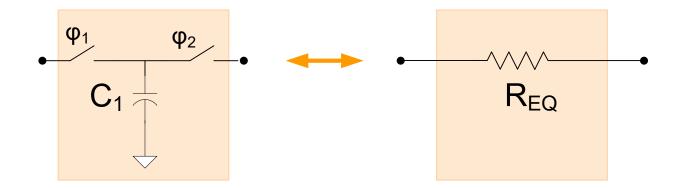
Observe that a switched-capacitor behaves as a resistor!

This is an interesting observation that was made by Maxwell over 100 years ago but in and of itself was of almost no consequence

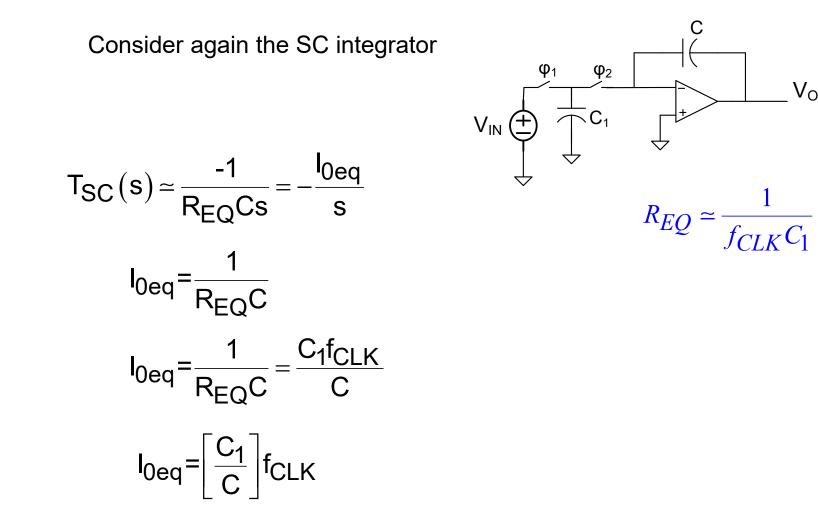
Note that large resistors require small capacitors !

This offers potential for overcoming <u>one</u> of the critical challenges for Implementing integrators on silicon at audio frequencies!

Equivalence Between Rapidly Switched Capacitor and Resistor



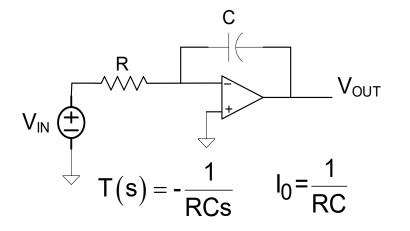
$$R_{EQ} \simeq \frac{1}{f_{CLK}C_1}$$

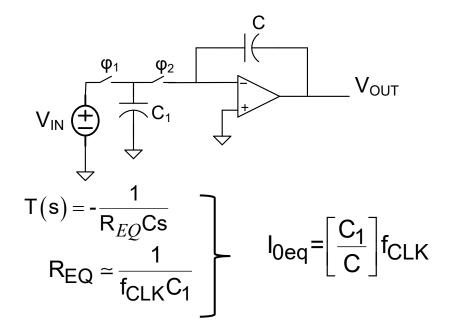


V_{OUT}

This is a frequency referenced filter!

Consider again the SC integrator

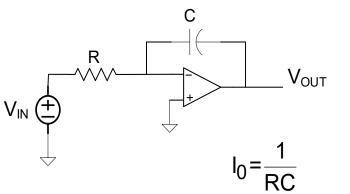




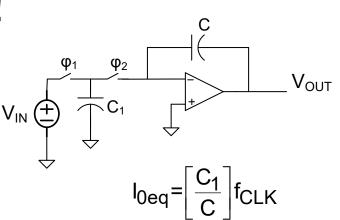
- 1. Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)
- 2. Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)
- 3. Amplifier GB limits performance

- 1. Accuracy of cap ratio and f_{CLK} very good
- 2. Area of C1 and C not too large
- 3. Amplifier GB limits performance less

The Genius !!



- 1. Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)
- 2. Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)
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- 1. Accuracy of cap ratio and f_{CLK} very good
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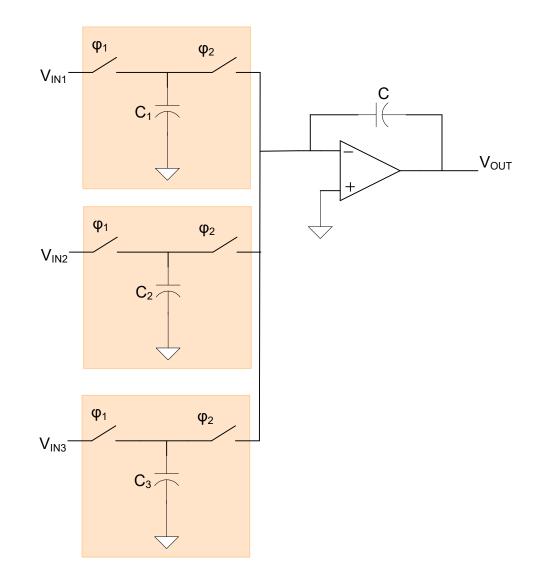
Observation of Maxwell (and other "Me Too" up until 1977) on equivalence of resistor and switched capacitor had no impact

Two groups independently observed items 1) and 2) in 1976/1977 timeframe and realized that practical implementations on silicon were possible and that is the genius of the concept

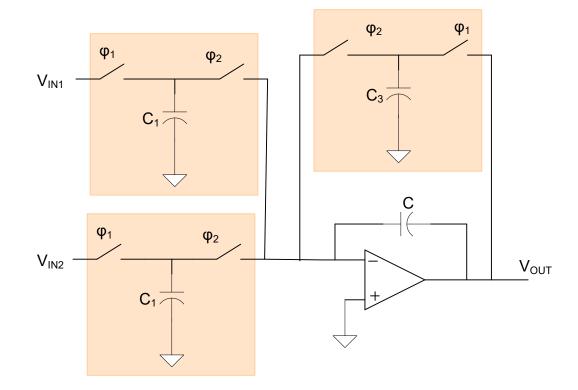
Switched Capacitors and the corresponding charge redistribution circuits now used well beyond the SC filter field

Incredible enthusiasm and effort followed for better part of a decade

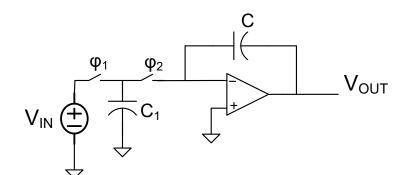
sC integrator with summing inputs

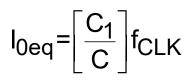


sC low-pass filter with summing inputs

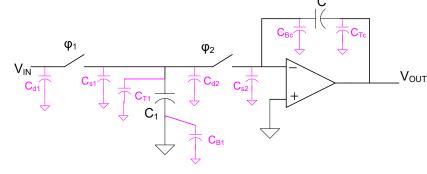


Consider again the SC integrator





Observe this circuit has considerable parasitics

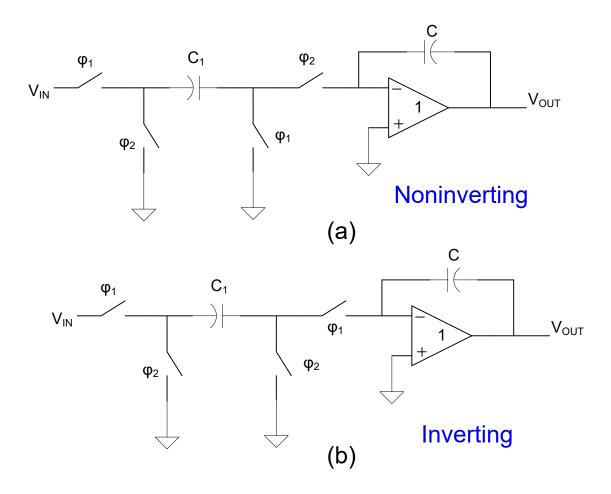


 $C_{1EQ} = C_1 + C_{s1} + C_{d2} + C_{T1}$

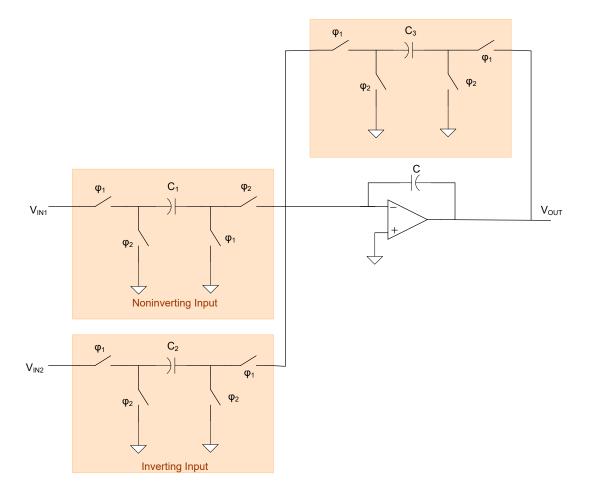
Parasitic capacitors $C_{s1}+C_{d2}+C_{T1}$ difficult to accurately match

- Parasitic capacitors of THIS SC integrator limits performance
- Other SC integrators (discussed next) ofter same benefits but are not affected by parasitic capacitors

Stray insensitive Inverting and Noninverting SC integrators

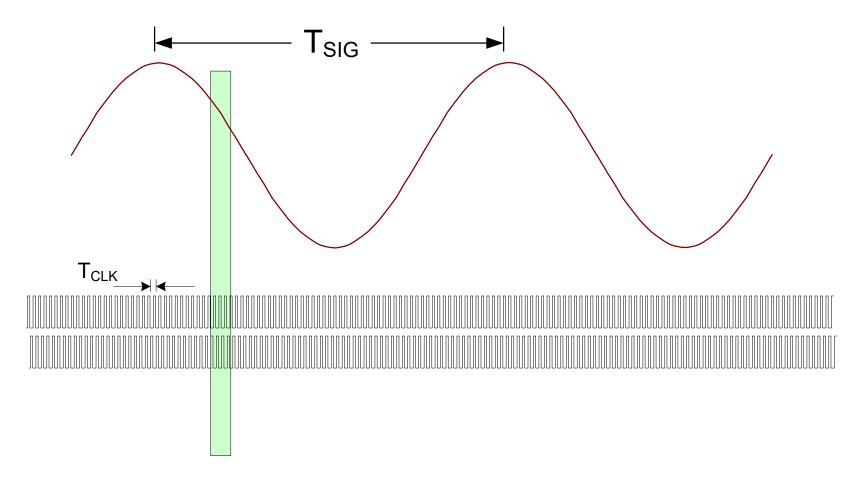


Stray Insensitive SC Low-Pass Filter with Inverting and Noninverting Inputs



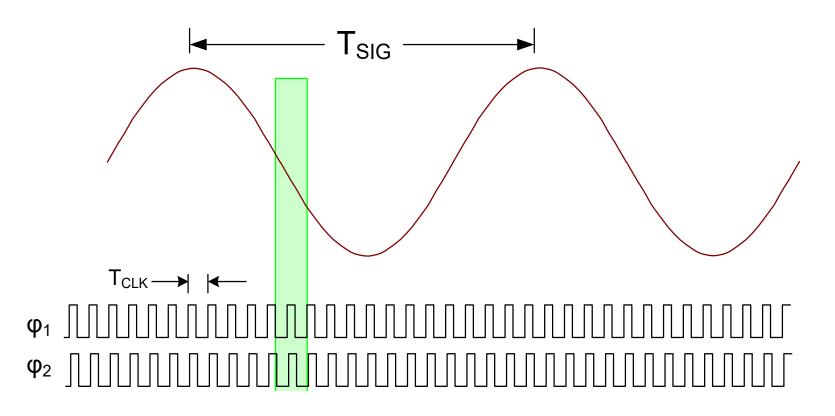
Arbitrary number of inverting and ioninverting Inputs can be added

What if T_{CLK} is not much-much smaller than $T_{SIG}?$ For $T_{CLK}{<<}T_{SIG}$



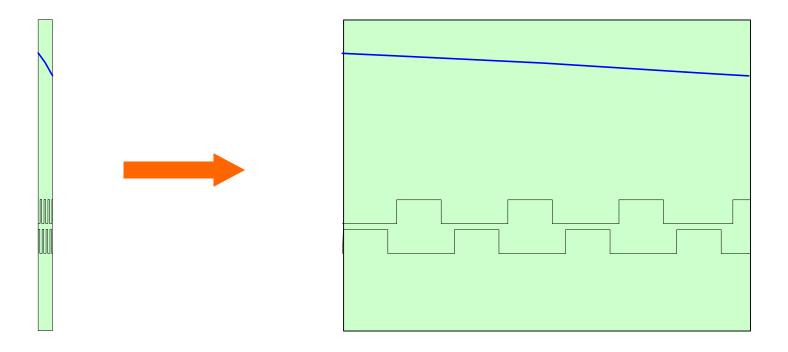
What if T_{CLK} is not much-much smaller than T_{SIG} ?

For T_{CLK} <T_{SIG}

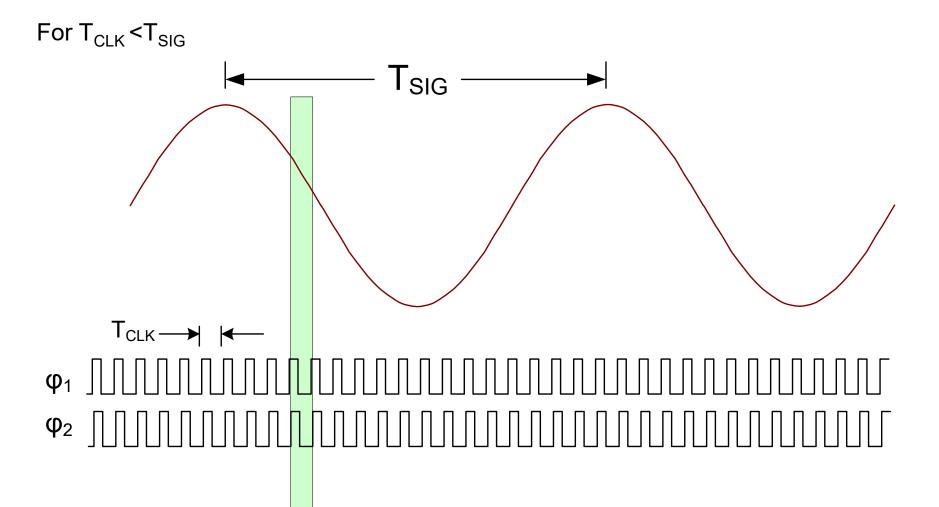


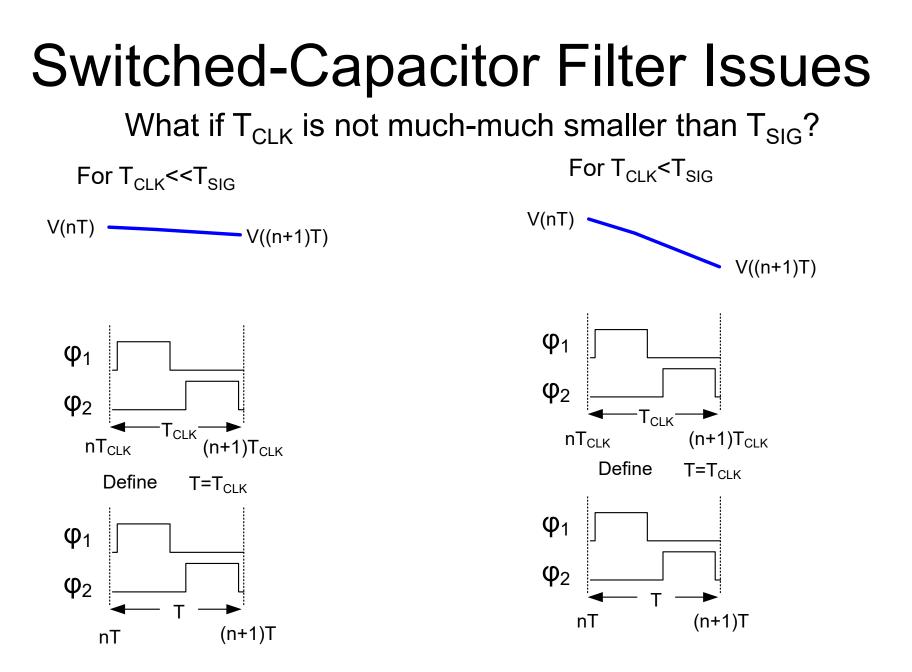
What if T_{CLK} is not much-much smaller than T_{SIG} ?

For T_{CLK}<<T_{SIG}

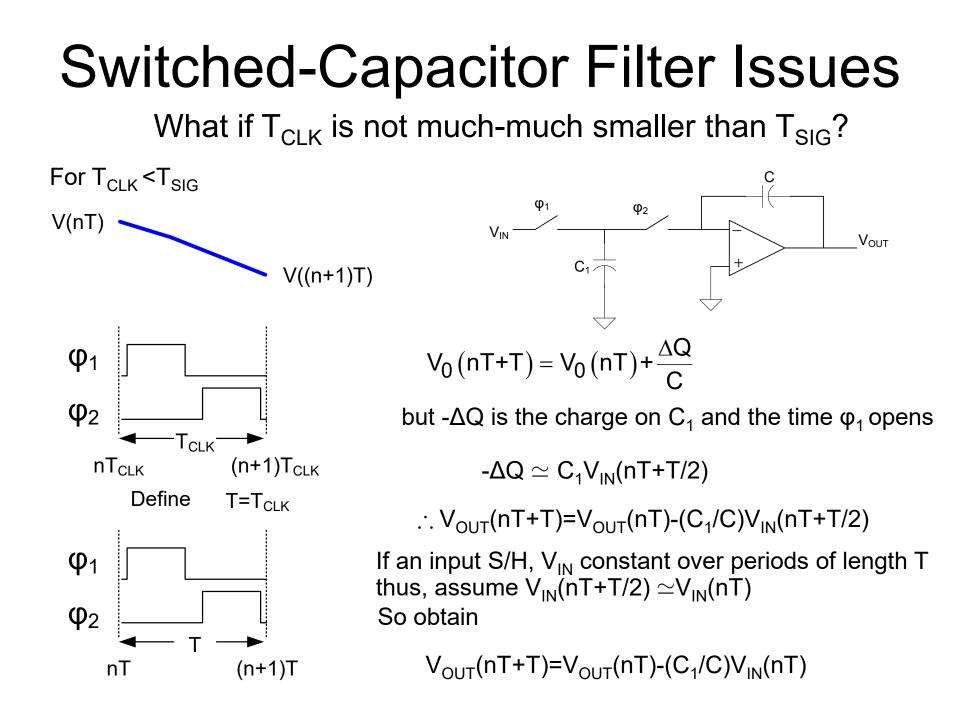


What if T_{CLK} is not much-much smaller than T_{SIG} ?

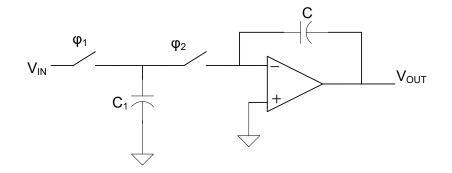




Considerable change in V(t) in clock period



What if T_{CLK} is not much-much smaller than T_{SIG} ?



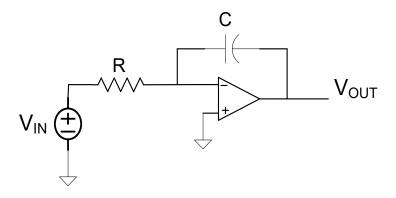
 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$

for any T_{CLK}, characterized in time domain by difference equation

or in frequency domain characterized by transfer function obtained by taking z-transform of the difference equation

$$H(z) = -\frac{\frac{C_1}{z}}{\frac{z}{z-1}}$$

What is really required for building a filter that has high-performance features?



Frequency domain:

Transfer function

 $T(s) = \frac{1}{RCs}$

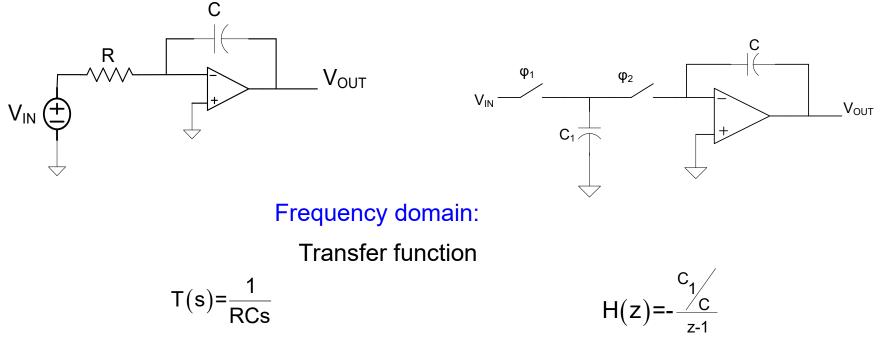
Time domain:

Differential Equation

$$V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^t V_{IN}(\tau) d\tau$$

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential equation

What is really required for building a filter that has high-performance features?



Time domain:

Differential Equation

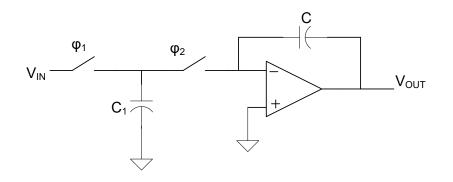
Difference Equation

$$V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^t V_{IN}(\tau) d\tau$$

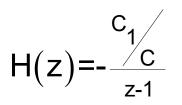
 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential/difference equation

What if T_{CLK} is not much-much smaller than T_{SIG} ?



 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$

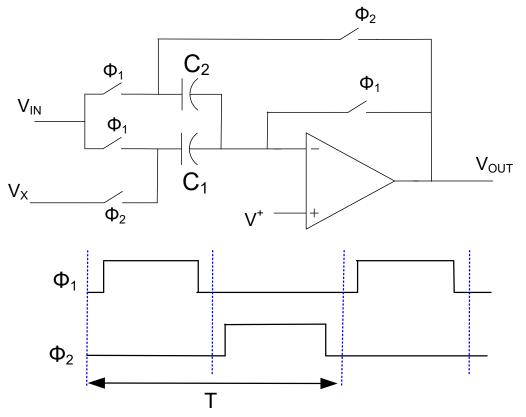


Switched-capacitor circuits have potential for good accuracy and attractive area irrespective of how T_{CLK} relates to T_{SIG}

But good layout techniques and appropriate area need to be allocated to realize this potential !

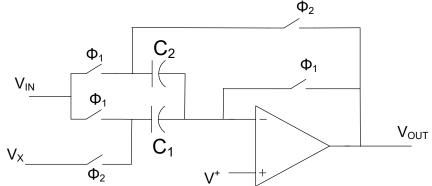
Consider the following circuit

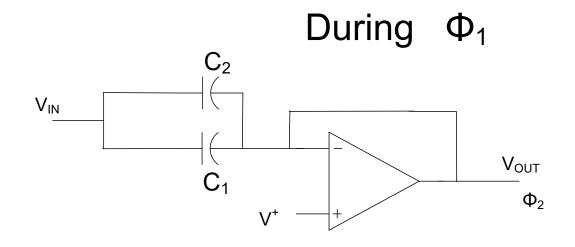
Termed a flip-around amplifier

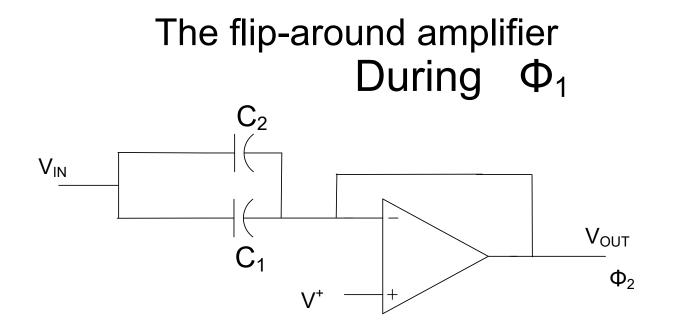


Clock signals are complimentary non-overlapping

The flip-around amplifier $_{\Phi_2}$

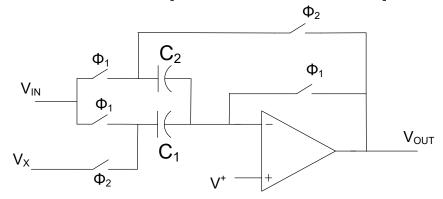




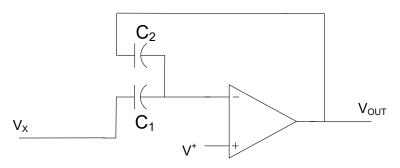


$$\mathbf{Q}_{1} = \mathbf{C}_{1} \left(\mathbf{V}_{\mathrm{IN}} - \mathbf{V}^{+} \right)$$
$$\mathbf{Q}_{2} = \mathbf{C}_{2} \left(\mathbf{V}_{\mathrm{IN}} - \mathbf{V}^{+} \right)$$

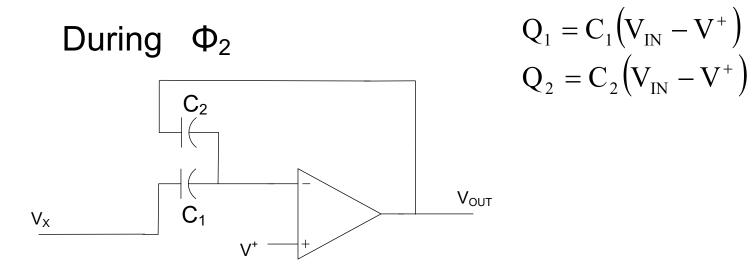
The flip-around amplifier

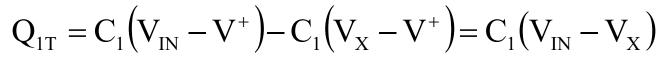


During Φ_2



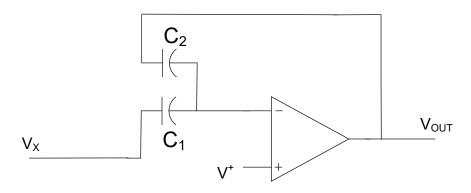
The flip-around amplifier





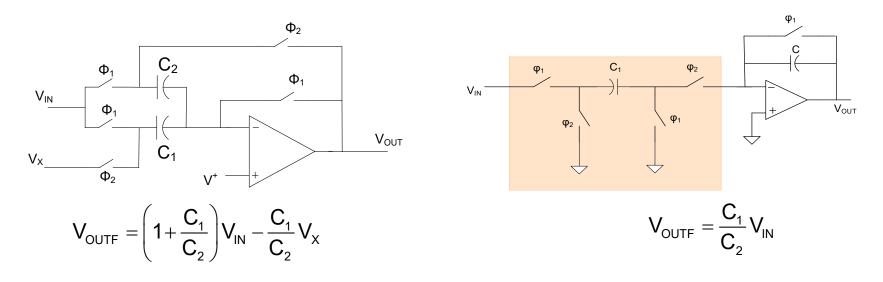
$$Q_{2F} = Q_2 + Q_{1T} = C_2 (V_{1N} - V^+) + C_1 (V_{1N} - V_X) = (C_1 + C_2) V_{1N} - C_2 V^+ - C_1 V_X$$

The flip-around amplifier During Φ_2



 $Q_{2F} = Q_{2} + Q_{1T} = C_{2} (V_{IN} - V^{+}) + C_{1} (V_{IN} - V_{X}) = (C_{1} + C_{2})V_{IN} - C_{2}V^{+} - C_{1}V_{X}$ $V_{C2F} = \frac{Q_{2F}}{C_{2}} = \left(1 + \frac{C_{1}}{C_{2}}\right)V_{IN} - V^{+} - \frac{C_{1}}{C_{2}}V_{X}$ $V_{OUTF} = V_{C2F} + V^{+} = \left(1 + \frac{C_{1}}{C_{2}}\right)V_{IN} - \frac{C_{1}}{C_{2}}V_{X}$

Comparison of Flip Around Amplifier with previous SC amplifier



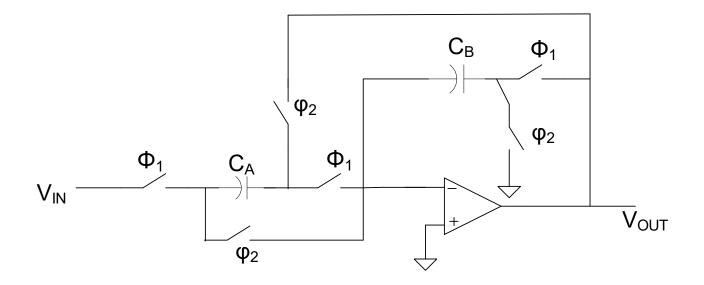
If $V_X=0$, both have a positive gain but somewhat more gain for a given capacitor ratio for the flip-around structure

In both cases, gain accuracy dependent upon how closely the capacitor ratios can be controlled

One particularly useful application is where want dc gain equal to 2 (1bit/stage pipeline ADC)

Flip-around requires matching two capacitors, other requires ratio matching of two capacitors

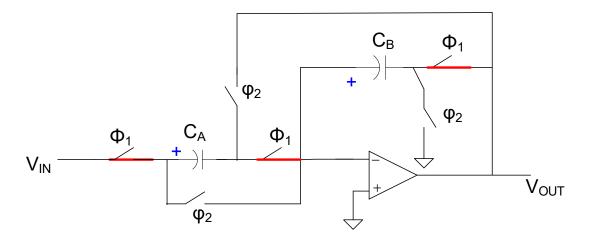
Another Flip Around Amplifier



Clock signals are complimentary non-overlapping

Another Flip Around Amplifier

During phase ϕ_1



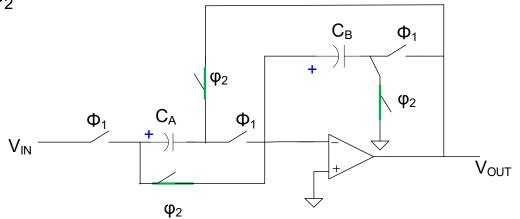
Assume C_B discharged at start of phase – must verify later

$$\begin{split} \boldsymbol{Q}_{\text{CA1}} &= \boldsymbol{C}_{\text{A}}\boldsymbol{V}_{\text{IN}} \\ \boldsymbol{Q}_{\text{CB1}} &= \boldsymbol{C}_{\text{A}}\boldsymbol{V}_{\text{IN}} \end{split}$$

$$V_{\text{OUT}} = -\frac{Q_{\text{CB1}}}{C_{\text{B}}} = -\frac{C_{\text{A}}}{C_{\text{B}}}V_{\text{IN}}$$

Another Flip Around Amplifier

During phase ϕ_2



From phase ϕ_1 $Q_{CA1} = C_A V_{IN}$ $Q_{CB1} = C_A V_{IN}$

$$Q_{CA2} = Q_{CA1} + Q_{CB1}$$
$$Q_{CB2} = 0$$
$$V_{OUT} = -\frac{Q_{CA2}}{C_A}$$
$$V_{CB} = 0$$

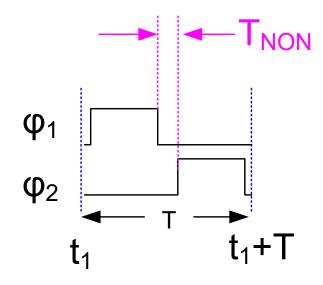
Verified that C_B was discharged at the start of phase ϕ_1

$$V_{\text{OUT}} = -\frac{C_{\text{A}}V_{\text{IN}} + C_{\text{A}}V_{\text{IN}}}{C_{\text{A}}} = -2V_{\text{IN}}$$

This structure has a gain of 2 independent of any capacitor matching!

Can modify to get noninverting gain and gains of 3, 4, .., without matching requirements

Non-overlapping Clocks



- Essential that the clocks be non-overlapping
- Simple inverter to derive the complimentary clock will not work
- Must guarantee non-overlap in the presence of PVT variations
- In non-demanding speed applications, ϕ_1 and ϕ_2 will have 25% duty cycles



Stay Safe and Stay Healthy !

End of Lecture 40